VOLUME 81 SEPARATE No. 586

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

JANUARY, 1955

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

MULTIPLE SPAN GABLED FRAMES

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STRUCTURAL DIVISION

(Discussion open until May 1, 1955)

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33 W. 39th St.
New York 18, N. Y.

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This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

MULTIPLE SPAN GABLED FRAMES

John D. Griffiths,[1] M. ASCE

INTRODUCTION

Multiple span frames of the type shown in Fig. 1 have long been recognized as efficient and economical, and they certainly provide a most acceptable architectural form. However, in spite of their many attributes, they have not enjoyed the wide popularity of single span frames. Their limited use has been due, in no small measure, to the complexity of their analysis and design.

The following material outlines a procedure of analysis which has been found to be less laborious than methods presently in common use. This procedure, an Analysis By Parts, does not require the solution of simultaneous equations and reasonable accuracy of results may be obtained by slide rule calculation. The presentation of this material presupposes a knowledge on the part of the reader of the basic concepts and terminology of the Hardy Cross Method of Moment Distribution.[1]

General Considerations

The great labor involved in the analysis of multiple span gabled frames is due primarily to the requirement for solution of difficult types of simultaneous equations. Too often, the simultaneous equations which express the relationship of joint translation (or sidesway) converge quite slowly. This lack of convergence of joint translation equations is in contrast to the rapid convergence of equations which express only joint rotation. The latter type are readily solved by moment distribution of by other procedures of iteration.[2]

A primary reason for the marked difference in the rates of convergence of the two types of equations can be realized from a consideration of moment and thrust carry over factors. The moment carry over factor is ordinarily 1/2, and thus, each cycle of distribution reduces fixed end moments remaining from the previous cycle by more than 50%. On the other hand, thrusts are carried over from one joint to the next at their full numerical value in order that ΣH may remain equal to zero. Therefore, the thrusts (associated with joint translations) tend to remain in the structure and will proceed toward the supports at only a slow pace under the pressure of a balancing procedure. This is particularly true in hinged base gabled frames where columns are relatively flexible.

It is possible to "distribute" thrusts throughout a structure in a single operation by use of a spring analogy as discussed by Dr. Francis.[3] However, this operation can be used only for pure translation (without rotation). Therefore this direct thrust distribution produces new moments which must be distributed and which produce in turn new thrusts, etc.

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1. Chf. Engr., Gate City Steel, Inc., Boise, Idaho.

Small superscript numbers 1 through 11 refer to entries in the Bibliography.

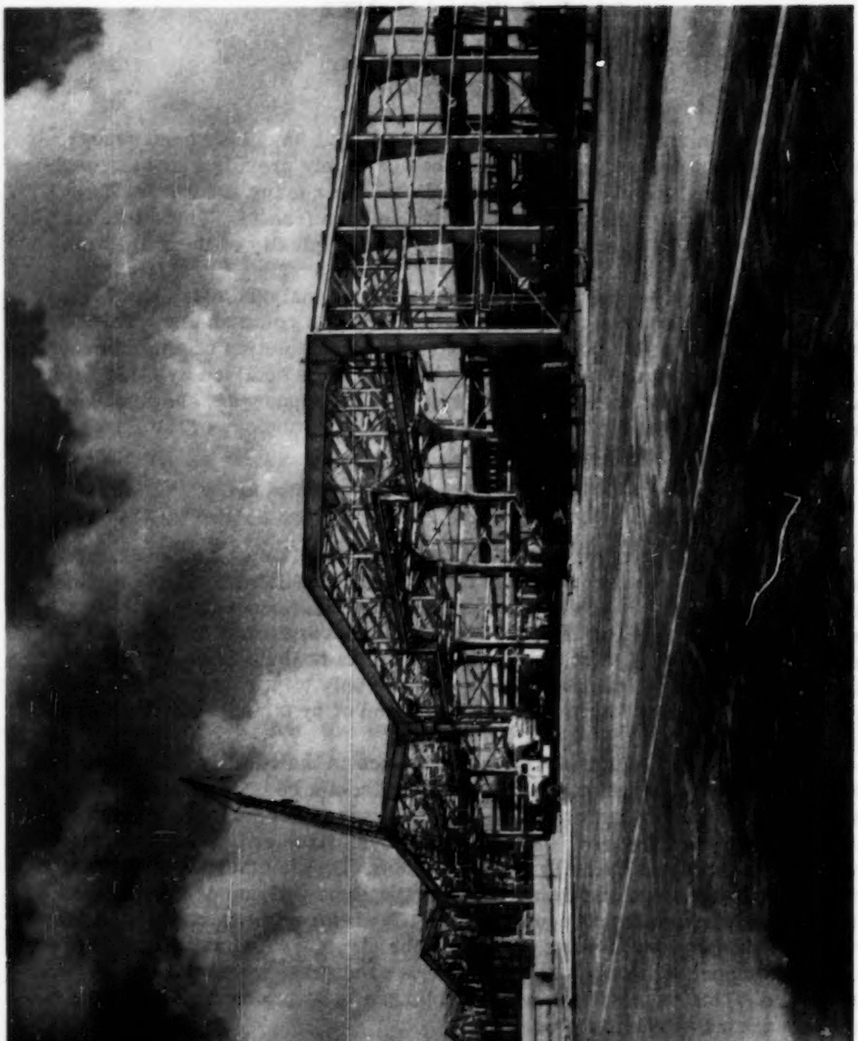


Fig. 1 Philadelphia International Airport Hangars.

Equations which will yield a direct final solution must express combined joint rotation and translation relationships. Unfortunately, these equations inherit much of the tendency for slow conversion from the expressions of pure translation. Such equations may result from a classical analysis such as slope deflection or from the sidesway correction operations of the method of moment distribution.

It is desirable to eliminate, on one hand, the requirement for solution of complicated simultaneous equations and to avoid, on the other hand, the necessity for successive cycles of moment-thrust distribution.

Sign Convention

The sign convention as listed below and as indicated in Fig. 2. will be used in this paper.

Clockwise rotations are positive
Clockwise moments are positive
Translations to the right are positive
Thrusts to the right are positive

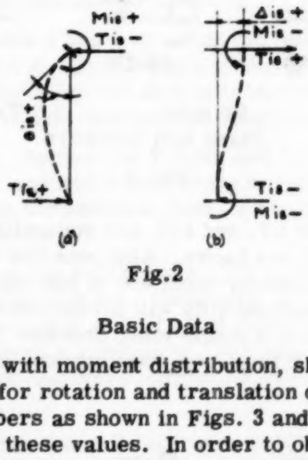


Fig.2

Basic Data

The reader, familiar with moment distribution, should recognize the values of moments and thrusts for rotation and translation of pinned and fixed end straight prismatic members as shown in Figs. 3 and 4. However, it is not enough to just recognize these values. In order to obtain accuracy and speed in the method outlined, all values, with proper sign, must be instantly on the tip of the tongue regardless of the position of the member and regardless of which end of the member is to be rotated or translated. If any term is not familiar to the reader, the derivation of that term should, of course, be reviewed in one of the standard textbooks on indeterminate structures.[4,5,6]

Because only relative values of rotation and translation are required, the term E has been omitted in Figs. 3 and 4. For exact values, each term associated with rotation and translation should be multiplied by E.

The following system of notation is used in Figs. 3 and 4 and in the discussions of this paper. However, in the actual analysis of frames, all moments will be designated simply as M regardless of the cause for their existence. Likewise, all thrusts will be designated simply as T.

F.E.M. = fixed end moment
F.E.T. = fixed end thrust

$\overline{M}$ = rotation stiffness

M' = moment carried over to the far end of a member by a unit rotation of the near end

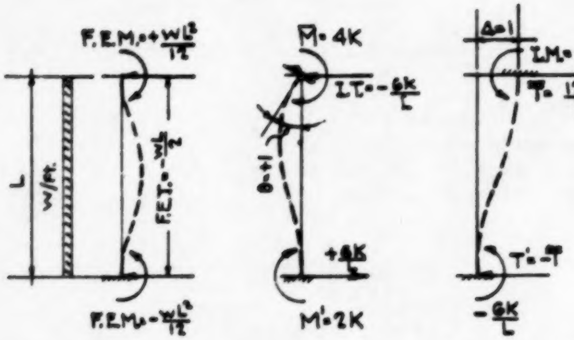
I.T. = thrust induced by a unit rotation

$\overline{T}$. = translation stiffness

T' = thrust carried over to the far end of a member by a unit translation of the near end

I.M. = moment induced by a unit translation

K = I/Length of member

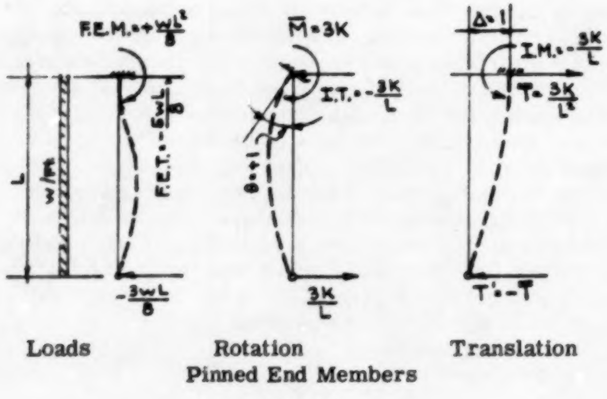


Loads Rotation Translation

Fixed End Members

Fig. 3

Note that the values of I.T. and I.M. are statically determinate once the values of $\overline{M}$, M' or $\overline{T}$, T' are known. Also note that for any member of I.M. and I.T. values are numerically equal and of like sign. It has been shown by Niles and Newell[7] that such equality will always exist for any member or combination of members at a single joint, provided that the induced moments and induced thrusts result from unit rotations and unit translations.



Loads Rotation Translation

Pinned End Members

Fig. 4

Example 1

Figure 5 provides an analysis, in tabular form, of a two member frame. This frame will later be considered as a part of a more complex structure. In the tabular solution of Figure 5, all moments are designated simply as M and all thrusts as T even though such moments and thrusts may be due either to joint rotations, joint translations or load applications. Calculations are, in general, to an accuracy of three significant figures. A detailed description of the analysis follows:

Step 1 Line 1: Record moments and thrusts associated with a positive unit rotation of end A of Member AK (I.E. θ AK). The moment at A is $\overline{M}$ = 4K = 4 x 40 = 160. The thrust at A is I.T. = 6K/L = -6 x 40/12 = -20.

Step 2 Line 2: Record moments associated with a positive unit rotation of end A of member AB. The moments are $\overline{M}$ = 4K = 4 x 10 = 40 at A and M' = 2K = 2 x 10 = 20 at B. The thrusts at A and B due to this rotation of A will be equal and opposite ($\Sigma H = 0$). Therefore, their combined effect on the frame reactions and on frame deformations is zero (deformations due to direct stress are disregarded). For these reasons, the thrusts associated with this operation are not required and are therefore not recorded in the tabular solution.

Step 3 Line 3: Add lines 1 and 2 to obtain the moments and thrusts for a unit rotation of joint A. On the basis of the same reasoning as outlined under step 2, the thrust of -20 at A is transferred directly to B. The frame reactions and deformations are unaffected thereby.

Step 4 Lines 4, 5 and 6: Record the F.E.M. and F.E.T. due to loads for each individual member and add these values to obtain the total F.E.M. and F.E.T. Note that in the addition (Line 6) the thrust at A(-80) is again transferred directly to B.

Step 5 Line 7: Balance the F.E.M. at A (107 ft. kips) by the application of an equal and opposite moment. Obviously, if + 200 ft. kips (line 3) is associated with a rotation at A of + 1.0 units, a moment of -107 ft. kips will be associated with a rotation of -107/200 = -.535 units. All of the figures of line 3 are therefore multiplied by -.535 and the results are recorded on line 7.

Step 6 Line 8: Add lines 6 and 7. If frame BAK were not to be later considered as a part of a larger structure, line 8 would complete the analysis.

It has now been determined, that under the action of the given loads, joint A will rotate -.535 units (relatively) and the reactions at B will be M = + 2.6 ft. kips and T = -89.3 kips. These values of moment and thrust may be considered as values of F.E.M. and F.E.T. for member BK on the assumption that the frame BAK acts as a single member with joint A eliminated, or at least temporarily disregarded.

If BK is a part of a more complex frame which extends beyond joint B, it will be desirable to determine its rotation and translation stiffness at joint B. This can be done as indicated below.

Step 7 Line 9: Record moments associated with a unit rotation of joint B. As indicated under Step 2, it is not necessary to record thrusts in this case.

Step 8 Lines 10 and 11: Balance moments at A as outlined under Steps 5 and 6. The moment of + 38 ft. kips is the moment stiffness of BK (i.e. it is the moment associated with a unit rotation of Bk, joint A being relaxed). The thrust of + 2 kips is the induced thrust for BK.

Step 9 Lines 12 through 14: Translate joint B one unit. Note that as joint B is translated (without vertical movement of that joint and without rotation of joint A) no bending deformation occurs in member BA. Also note that the entire thrust applied at B is transmitted to A. Therefore, the thrust required for unit translation of joint B is equal to the thrust required for unit translation of end A of the member AK (i.e.

$\overline{T} = \frac{12K}{L^2} = 3.33$ and I.M. $= -\frac{6K}{L} = -20$).

Now record the moments and thrust associated with this translation and balance joint A in accordance with procedures previously outlined. The translation stiffness (+ 1.33) and the induced moment (+ 2.0) for BK have now been determined.

The total end moment for any member of a structure will always be the sum of the following three values:

1. The original fixed end moment
2. The moment due to joint rotations
3. The moment due to joint translations

Let it now be assumed that joint B, due to action of the frame extending beyond that joint, is rotated + 2 units and translated -3 units. The total moments in the frame will now be determined on the basis of this arbitrary assumption.

Step 10 Lines 15 thru 24: Figure 5 provides the following values for BK:

F.E.M. (or loads BK) = + 2.6 (line 8)
Rotation stiffness = + 38 (line 11)
I.M. = + 2 (line 14)

Therefore:

$M_{BK} = + 2.6 + 2 \times 38 - 3 \times 2 = + 72.6$ ft. k.

Similarly:

$T_{BK} = -89.3 + 2 \times 2 - 3 \times 1.33 = -89.3$ k.

Figure 5 also shows that:

θ_A due to loads = -.535 (line 8)
θ_A due to θ_B = -.1 (line11)
θ_A due to Δ_B = + .1 (line 14)

Therefore:

Total θ_A = -.535 -2 x .1 -3 x .1 = 1.04

The translation of joint A is obviously equal to the translation of joint B (i.e. $\Delta_A = -3$).

Figure 5 provides the following values for AK:

F.E.M. = + 120 (line 4)
Rotation stiffness = + 160 (line 1)
I.M. = - 20 (line 12)

Therefore:

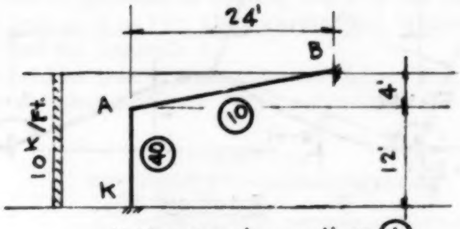
$M_{AK} = + 120 - 1.04 \times 160 + 3 \times 20 = + 14$ ft. k.

Similarly:

$T_{AK} = -60 + 1.04 \times 20 - 3 \times 3.33 = -49.2$ k.

With the above moments and thrusts determined, all other moments and reactions of the frame may be obtained by statics.

It would be well for the reader, at this stage, to review the above description until he is able to set down the tabulation of Fig. 5 without reference to written material and without the use of any scratch paper.



K Values shown thus (1)

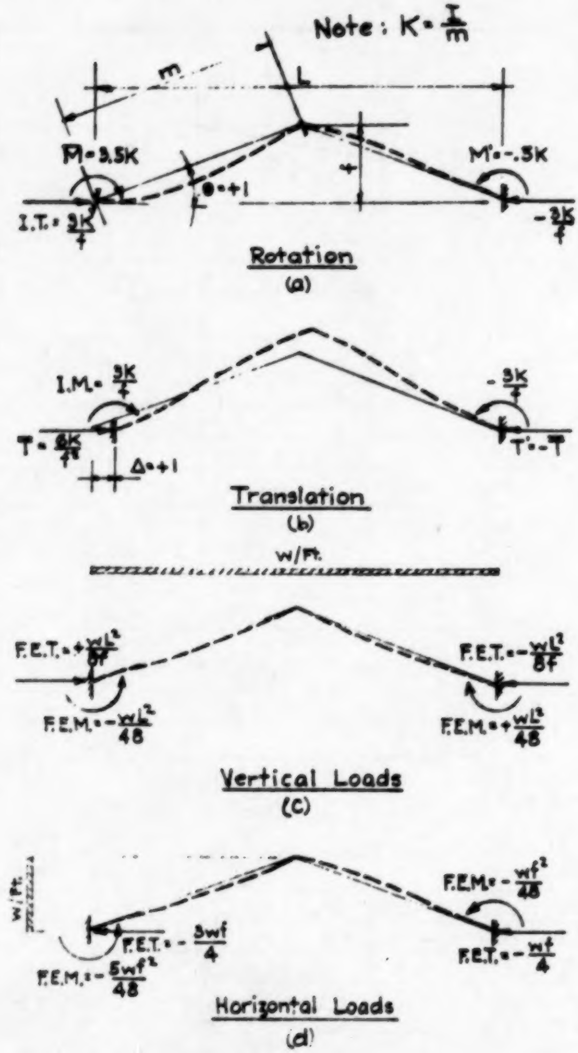
		Joint A		Joint A		Joint B	
		θ	Δ	M	T	M.	T
1	θAK			+160	-20		
2	θAB			+ 40		+20	
3	Total	+1.0		+200		+20	-20
4	Lds AK			+120	-60		
5	" AB			-13.3	- 20	+13.3	-20
6	"Total			+107		+13.3	-100
7	Bal. M	-.535		-107		-10.7	+10.7
8	Lds BK	-.535		0		+2.6	-89.3
9	θBA			+20		+40	
10	Bal. M	-.1		-20		-2	+2
11	θBK	-.1		0		+38	+2
12	ΔBA			-20			+3.33
13	Bal. M	+.1		+20		+2	-2
14	ΔBK	+.1		0		+2	+1.33
15	BK	M	T	θB	ΔB	θA	ΔA
16	Loads	+2.6	-89.3	+2*		-.2	0
17	+2θBK	+76	+4		-3*	-.3	-3
18	-3ΔBK	-6	-4	Loads		-.535	0
19	Total	+72.6	-89.3	Total		-1.04	-3
20	AK	M	T				
21	Loads	+120	-6.0	*Assumed movements			
22	-1.04θAK	-166	+20.8	of joint B due to forces			
23	-3 ΔAK	+60	-10	not indicated in the			
24	Total	+14	-49.2	above sketch.			

Example 1

Fig. 5

Symmetrical Gables

Figure 6 provides general values of stiffness, etc. for symmetrical gables. The derivation of these values is outlined in the appendix.



Horizontal Loads

(d)

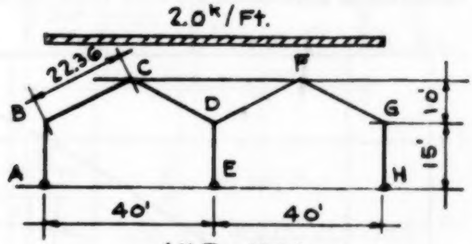
Symmetrical Gables

Fig. 6

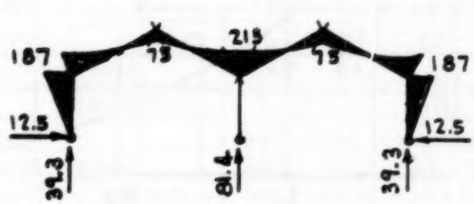
Example 2

The frame of Fig. 7 was analyzed in a recent publication,[8] by the method of work. The moment and reaction values obtained in Fig. 7 agree (within less than 1%) with the work solution. This frame is of such proportions that a rotation of joint B does not result in an induced thrust at that joint. Due to symmetry of frame and loading, no rotation or translation of the center joint D will occur. These conditions, of course, simplify the required solution. The calculations indicated in Fig. 7 are carried out in accordance with the principles described for Example 1.

The reader should be able to reproduce the data of Fig. 7 without reference to any written material except Fig. 6 and without the use of a scratch



	Joint B		Joint B		Joint D	
	θ	Δ	M	T	M	T
θBA			+200	-13.3		
θBD			+157	+13.4	-22.4	-13.4
Total	+1.0		+357	—	-22.4	-13.4
ΔBA			-13.3	+.89		
ΔBD			+13.4	+2.68	-13.4	-2.68
Total		+1.0	—	+3.57	-13.4	-2.68
Lds. DB			-66.7	+40	+66.7	-40
Bal. M	+.187		+66.7	0	-4.2	-2.5
Bal. T		-11.2	0	-40	+150	+30
Lds. DA	+.187	-11.2	0	0	+213	-12.5



Final Moments & Reactions

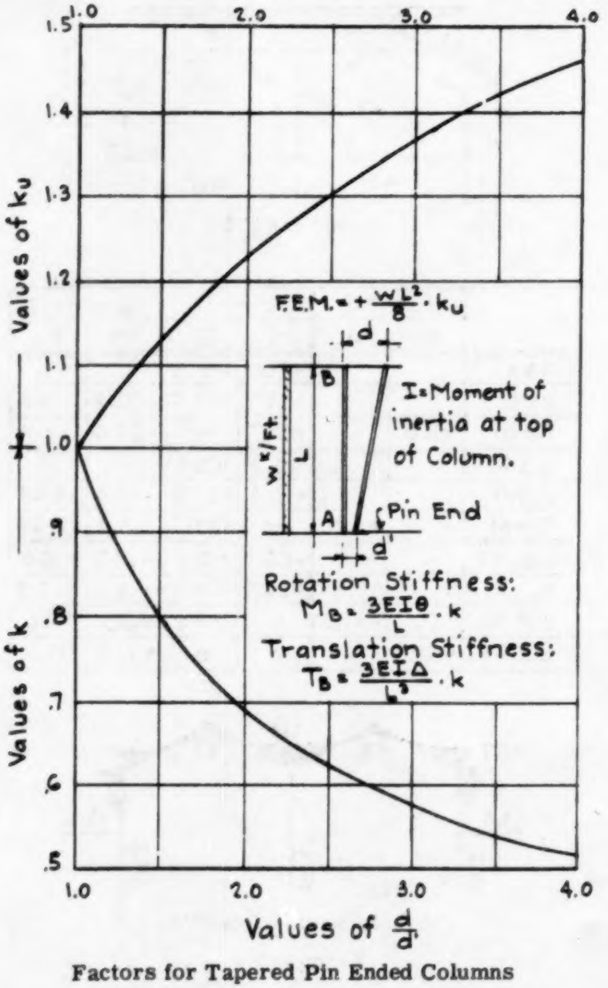
Example 2

Fig. 7

paper except for computations of statics required for the final moment diagram.

Variable Depth Members

It is generally economical to reinforce the frame members at the juncture of columns and gables by the use of a straight or curved haunch. The deepened section thus provided will cause increased moment at the haunch location. For a haunched knee of usual proportions, the calculated value of such moment increase will rarely exceed 5%. The actual moment increase may be even less than that calculated due to the non-uniform stress distribution across the knee. An investigation of a curved knee frame at Lehigh University[9] indicated a reduction of approximately 3.5% from the calculated knee



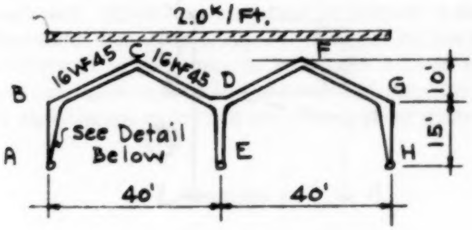
Factors for Tapered Pin Ended Columns

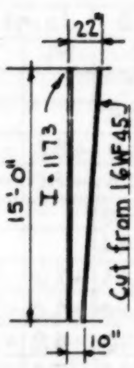
Fig. 8

moment due to the shift of the neutral axis toward the inner flange.

Some engineers entirely neglect the effect of a haunch; others neglect the haunch during analysis but, for design purposes, arbitrarily increase the uniform section knee moments by 3 to 5%.[10] In any case, it is generally agreed that an exact analysis of the effect of a typically proportioned haunch is rarely, if ever, justified.

The above observation regarding haunches of usual proportion does not





$$\frac{d}{d'} = \frac{22}{10} = 2.2$$

$$\theta_{BA}: M = \frac{3 \times 1173}{15} \times .67 = 157$$

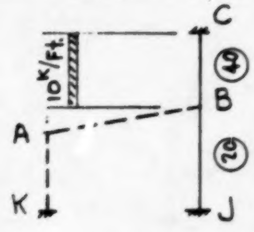
$$\Delta_{BA}: T = \frac{3 \times 1173}{15^3} \times .67 = .699$$

		Joint B		Joint B		Joint D	
		θ	Δ	M	T	M	T
1	θBA			+157	-10.5		
2	θBD			+91	+7.83	-13.0	-7.83
3	Total	+1.0		+248	-2.67	-13.0	-7.83
4	ΔBA			-10.5	+.699		
5	ΔBD			+7.83	+1.57	-7.83	-1.57
6	Total		+1.0	-2.67	+2.27	-7.83	-1.57
7	Bal. M	+.0108		+2.67	-.03	-.14	-.08
8		+.0108	+1.0	0	+2.24	-7.97	-1.65
9	Lds. DB			-66.7	+40	+66.7	-40
10	Bal. M	+.269		+66.7	-.72	-3.5	-2.1
11	Bal. T	-.189	-17.5	0	-39.3	+139	+28.9
12	Lds DA	+.080	-17.5	0	0	+202	-13.0

Example 3

Fig. 9

necessarily apply to the case of tapered members. A large variation in moment of inertia extending over a considerable portion of the frame may materially affect the moment and thrust distribution. Figure 8 provides factors for the rotation and translation stiffness of pin ended tapered sections. The curves shown are based on the assumption that I for a WF section varies as $(d)^{2,3}$.



		Joint B		Joint B		Joint C	
		θ	Δ	M	T	M	T
1	θBK			+38	+2		
2	θBJ			+80	-7.5		
3	θBC			+160	+20	+80	-20
4	Total	+1.0		+278	+14.5	+80	-20
5	ΔBK			+2	+1.33		
6	ΔBJ			-7.5	+.94		
7	ΔBC			+20	+3.33	+20	-3.33
8	Total		+1.0	+14.5	+5.60	+20	-3.33
9	Bal. M	-.0522		-14.5	-.76	-4.2	+1.04
10		-.0522	+1.0	0	+4.84	+15.8	-2.29
11	θCB			+80	+20	+160	-20
12	Bal. M	-.288		-80	-4.2	-23	+5.76
13	Bal. T	+.170	-3.26	0	-15.8	-51.5	+7.47
14	θCK	-.118	-3.26	0	0	+85.5	-6.77
15	ΔCB			-20	-3.33	-20	+3.33
16	Bal. M	+.0719		+20	+1.04	+5.75	-1.44
17	Bal. T	-.0247	+.473		+2.29	+7.47	-1.08
18	ΔCK	+.0472	+.473	0	0	-6.78	+.81
19	Lds. BK			+2.6	-89.3		
20	" BC			-120	-60	+120	-60
21	" Total			-117	-149	+120	-60
22	Bal. M	+.421		+117	+6.1	+33.7	-8.42
23	Bal. T	-1.54	+29.5		+143	+466	-67.6
24	Lds CK	-1.12	+29.5	0	0	+620	-136

Example 4

Fig. 10

Example 3

The frame of Example 2 is re-analyzed in Fig. 9 on the basis of tapered columns. It will be noted that the horizontal reactions at B and G increase from 12.5 kips for straight columns to 13.0 kips for the tapered columns. This is an increase of only 4% for this particular condition of frame geometry and loading.

All calculations for the tabular analysis of Fig. 9 are in accordance with steps previously outlined. However, attention is invited to the values of line 8 which are obtained from line 6 by a typical moment balance. By use of the values on line 8, thrusts at joint B may be balanced without inducing a moment at that joint. Line 8 provides values for a combined rotation and translation operation by which successive cylces of moment-thrust distribution may be avoided.

Example 4

The BAK portion of the frame of Fig. 10 was analyzed in Example 1. Therefore, the values on lines 1, 5 and 19 (Fig. 10) are copied directly from the result of Example 1. Each step in the present analysis is in accordance with previously explained procedures. The information obtained by the calculations of Fig. 10 will allow the frame CJK to be considered as a part of a still more complex structure.

Example 4 does not include the calculation of moments and thrusts at the ends of the individual members. These values may be determined if desired by the procedure described for lines 15 thru 24, Example 1.

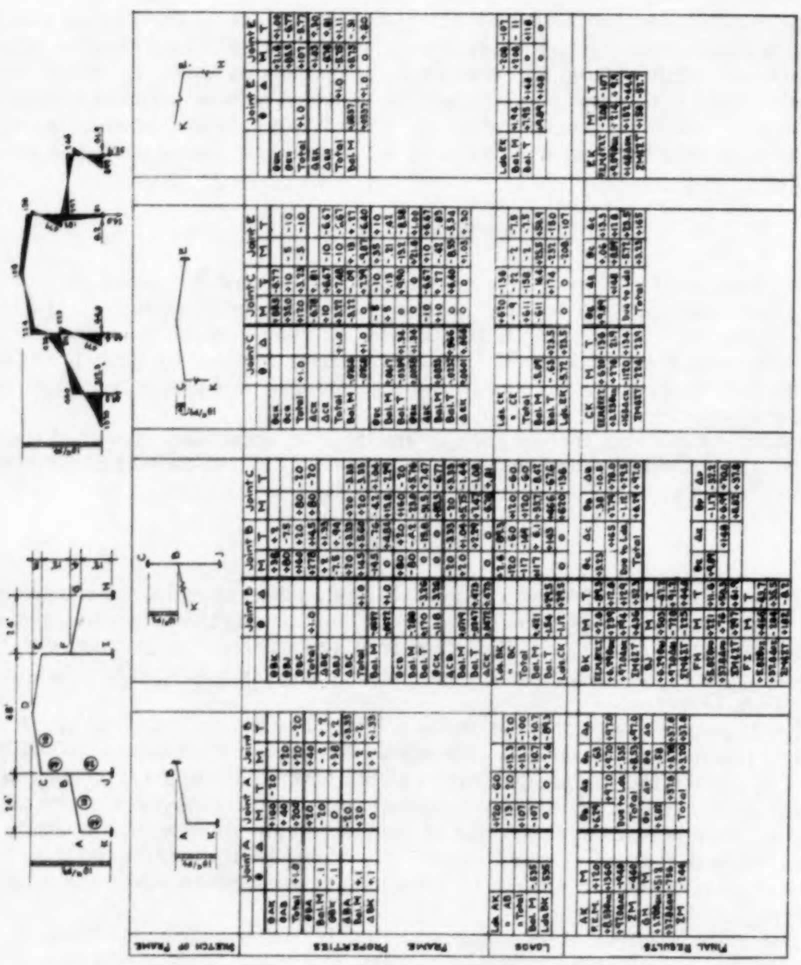
Example 5

In the "Analysis of Rigid Frames,"[2] a solution is provided for the structure of Fig. 11. A comparison of the results obtained in the referenced text with those obtained in Fig. 11 indicates good agreement. The greatest difference between the two solutions for any final moment is 8 ft. kips for M_{KA} (less than 1% error).

It will be observed that the solution of Fig. 11 is divided into four parts. Part 1, frame BAK, is the first example in this paper. Part 2, frame CJK (or just CK), is the fourth example. The calculations for these previous examples have been relisted in Fig. 11 with a slightly different arrangement of the tabulation. This revised arrangement is used to completely separate the calculations of frame properties and loads. If a second loading arrangement is to be later investigated, it will not be necessary to recalculate the frame properties.

The steps involved in the solution of Fig. 11 are in accordance with procedures already described. However, a few general comments regarding the calculations are in order. By symmetry of the frame, the moments and thrusts associated with a unit rotation or a unit translation of EH are numerically equal to those associated with a unit rotation or a unit translation of CK. These equalities are used in the analysis of joint E. Note that the determination of frame properties and loading effects proceeds from the left (joint A) to the right (joint E). The determination of final joint rotations and translations, on the other hand, proceeds from E toward A for the loaded portion of the frame and from E toward G for the unloaded portion.

Calculations are to an accuracy of three significant figures. All



Example 5

Figure 11

calculations for the indeterminate analysis, except direct slide rule operations, are listed. The computations of statics for the final moment diagram are not shown. No equations are used except for the relationships indicated in Figs. 3, 4 and 6.

CONCLUSION

As stated previously, the "Analysis by Parts," as described herein has been found less laborious than other methods for the design of Multiple Span Gabled Frames. Obviously, this procedure is applicable to the case of Wedge Beam Framing.[11] and its use may be extended to the case of more complex structures where joints have three instead of just two degrees of freedom.

The analysis of structures such as described herein, and the analysis of even more complex structures, brings to mind the final footnote in a textbook by Professor Linton Grinter. In part, Professor Grinter had this to say:

> "Before permitting the second volume of this series on structural theory to go into print, the author desires to emphasize his dissatisfaction with the theories presented . . ."

With such thoughts as the above in mind, it is encouraging to note the considerable research effort now being directed toward the study of plasticity. The several research programs on this subject hold out hope for substantial simplifications of analysis as well as a more rational approach to the design of continuous frames of ductile material.

APPENDIX

Figure 12(a) indicates a unit positive translation of end A of the unsymmetrical gable ABC. An enlargement of the deflection triangle $AA_2\,A_1$ is shown in Fig. 12(b). It is convenient to consider that the deflection AA_1 takes place in two steps; first, the deflection AA_2 (or Δ_3) which causes bending in member AB only; and second, the deflection $A_2\,A_1$ (or Δ_4) which causes bending in member BC only. From similar triangles:

$$x = \frac{\Delta_1 a}{f} = \frac{\Delta_2 b}{f}; \quad \frac{\Delta_1}{b} = \frac{\Delta_2}{a} = \frac{\Delta}{L}$$

$$\therefore\ \Delta_1 = \frac{\Delta b}{L} \quad \text{and} \quad \Delta_2 = \frac{\Delta a}{L}$$

$$\text{Also: } \Delta_3 = \frac{\Delta_1 m_1}{f} = \frac{\Delta b m_1}{fL}$$

$$\Delta_4 = \frac{\Delta a m_2}{fL}$$

The moment at each end of AB due to the deflection Δ_3 (Joint B restrained from rotation) is:

$$\text{I.M.} = +\frac{6K_1 \Delta_3}{m_1} = +\frac{6K_1\,\Delta}{f} \cdot \frac{b}{L}$$

Likewise, the moment at each end of BC due to the deflection Δ_4 (Joint B restrained from rotation) is:

$$I.M. = -\frac{6K_2\Delta}{f}\cdot\frac{a}{L}$$

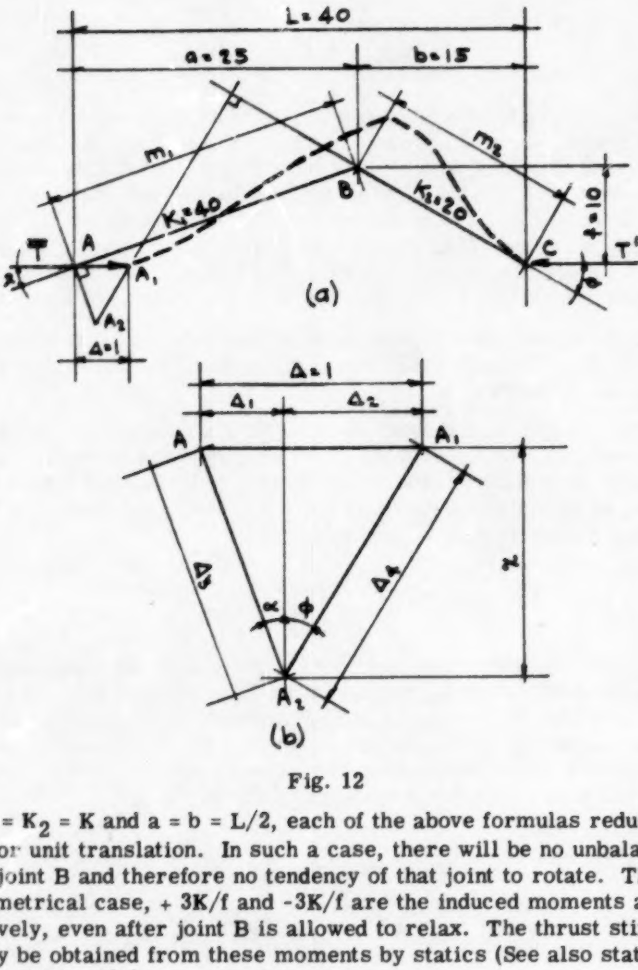


Fig. 12

If $K_1 = K_2 = K$ and $a = b = L/2$, each of the above formulas reduces to $\pm\ ^3K/f$ for unit translation. In such a case, there will be no unbalanced moment at joint B and therefore no tendency of that joint to rotate. Thus, for the symmetrical case, $+ 3K/f$ and $-3K/f$ are the induced moments at A and C respectively, even after joint B is allowed to relax. The thrust stiffness ($\overline{T}$) may be obtained from these moments by statics (See also statics calculations of Fig. 13b):

$$\Sigma M_B = +3K/f + 3K/f - Tf = O$$
$$\overline{T} = +6K/f^2$$

For the unsymmetrical gable of Fig. 12, the induced moments at each end of AB are:

$$+6K_1b/fL = +6 \times 40 \times 15/10 \times 40 = +9$$

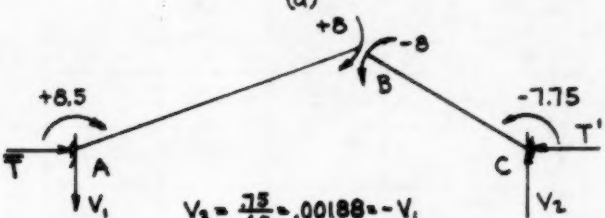
Likewise the induced moments at each end of BC are -7.5. Joint B may now be relaxed by moment distribution, as indicated in the table of Fig. 13 (a), to obtain the final induced moments. Subsequently, the thrust stiffness

may be determined by statics as indicated in Fig. 13(b).

In Fig. 14, values are obtained for moments due to uniform loads and unit rotation of a symmetrical gable. The thrusts associated with these moments may be obtained by statics as indicated in Fig. 13(b).

	Jt. B	Jt. A	Jt. B	Jt. C
	θ	M	M	M
θBA		+80	+160	
θBC			+80	+40
θB	+1.0	+80	+240	+40
Δ_3		+9	+9	
Δ_4			-7.5	-7.5
Total		+9	+1.5	-7.5
Bal. M	-.00625	-.5	-1.5	-.25
ΔAC		+8.5	0	-7.75

(a)



$$V_2 = \frac{.75}{40} = .00188 = -V_1$$

$$M_{BA} = +9 - .00625 \times 160 = +8$$

$$\Sigma M_B = 0 = +8 + 8.5 - .00188 \times 25 - 10T$$

$$T = +1.645$$

(b)

Fig. 13

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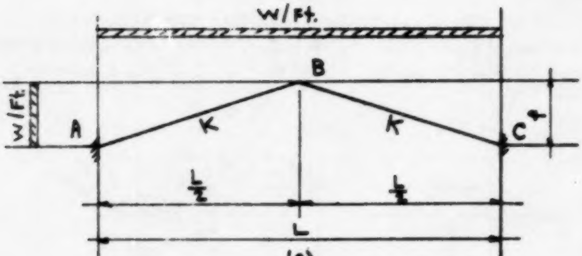

(a)

		Jt. B	Jt. A	Jt. B	Jt. C
		θ	M	M	M
Rotation	θBA		+2K	+4K	
	θBC			+4K	+2K
	θB	+1.0	+2K	+8K	+2K
	θAB		+4K	+2K	
	Bal.M	-.25	-.5K	-2K	-.5K
	θAC	-.25	+3.5K	0	-.5K
Vert. Loads	Lds. BA		$-\frac{wL^2}{48}$	$+\frac{wL^2}{48}$	
	" BC			$-\frac{wL^2}{48}$	$+\frac{wL^2}{48}$
	"Total " AC	0	$-\frac{wL^2}{48}$	0	$+\frac{wL^2}{48}$
Hor. Loads	Lds. BA		$-\frac{wf^2}{12}$	$+\frac{wf^2}{12}$	
	Bal. M	$-\frac{wf^2}{96K}$	$-\frac{wf^2}{48}$	$-\frac{wf^2}{12}$	$-\frac{wf^2}{48}$
	Lds. AC	$-\frac{wf^2}{96K}$	$-\frac{5wf^2}{48}$	0	$-\frac{wf^2}{48}$

(b)

Fig. 14

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